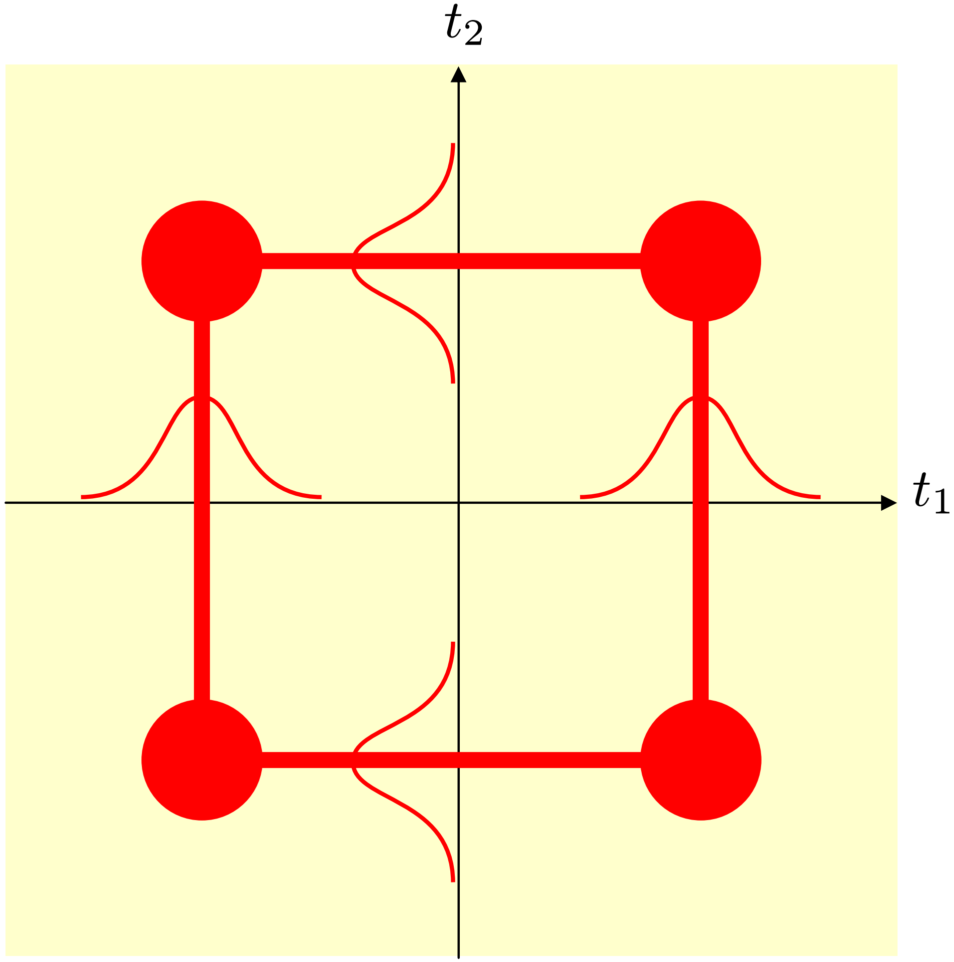
Funkcja Wignera

Funkcja spójności czasowej



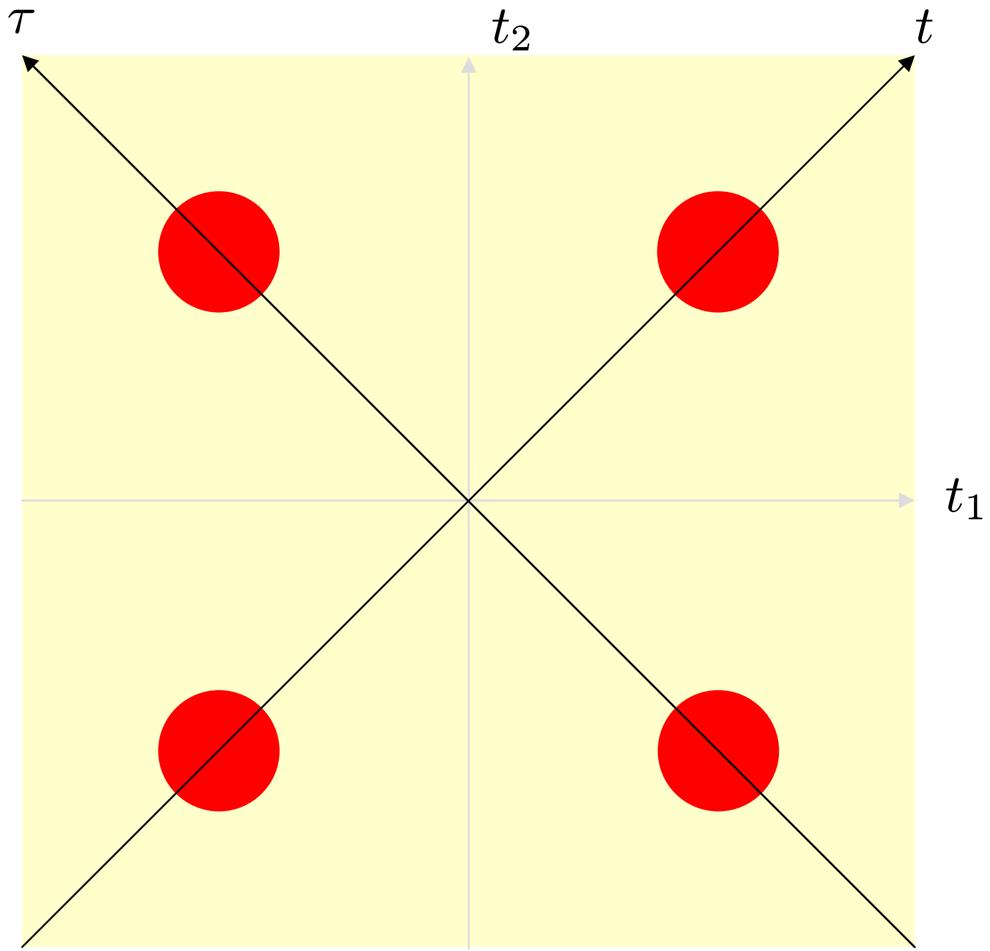
$$\langle \alpha(t_1)^* \alpha(t_2) \rangle$$

dwa spójne impulsy

$$I(t) =$$

$$\tilde{I}(\omega) =$$

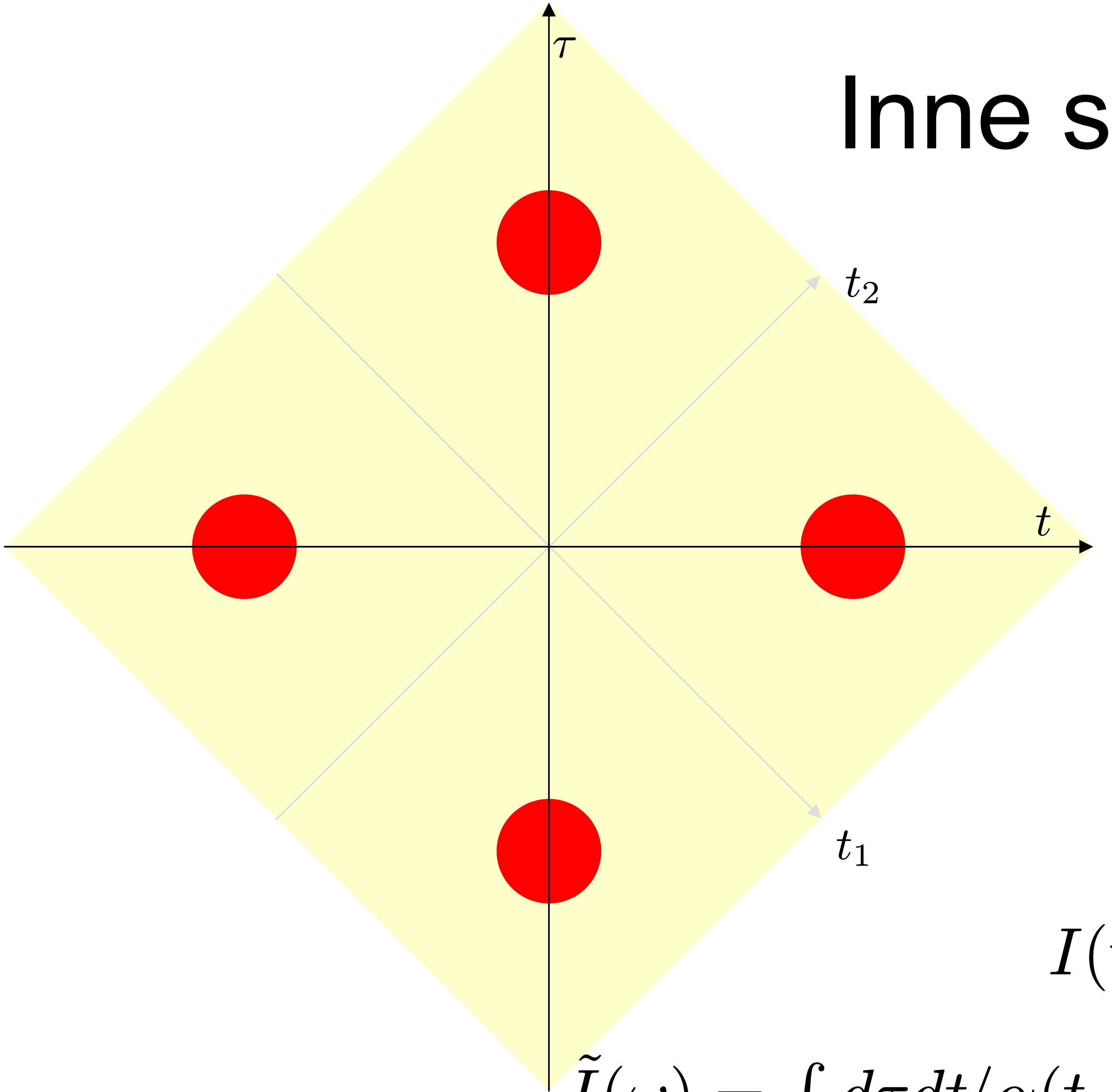
Kierunki funkcji spójności



$$I(t) = \langle \alpha(t)^* \alpha(t) \rangle$$

$$\tilde{I}(\omega) = \int dt_1 dt_2 \langle \alpha(t_1)^* \alpha(t_2) \rangle e^{-i\omega(t_2 - t_1)}$$

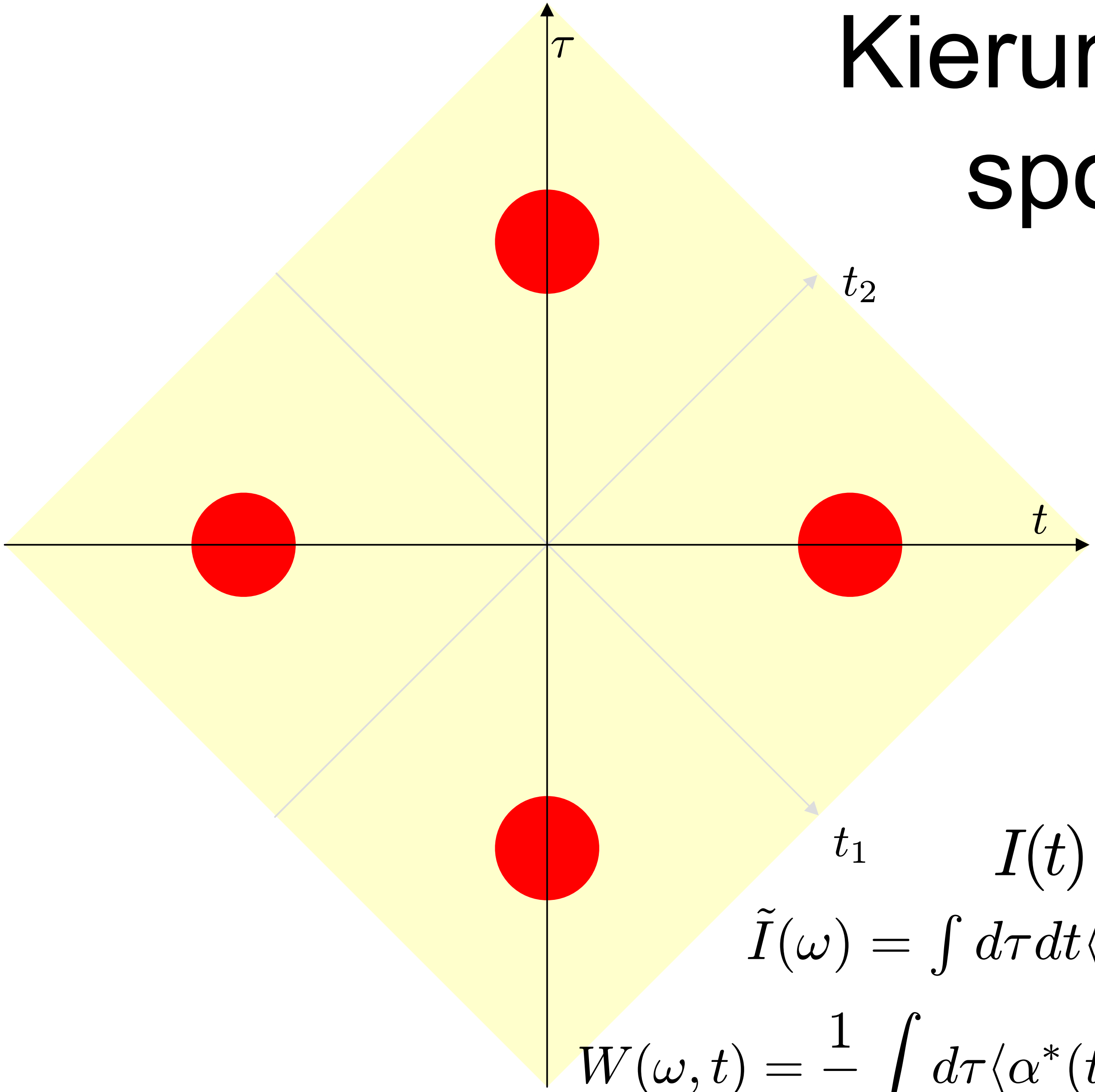
Inne spojrzenie



$$I(t) = \langle \alpha(t)^* \alpha(t) \rangle$$

$$\tilde{I}(\omega) = \int d\tau dt \langle \alpha(t_1)^* \alpha(t_2) \rangle e^{-i\omega(t_2 - t_1)}$$

Kierunki funkcji spójności

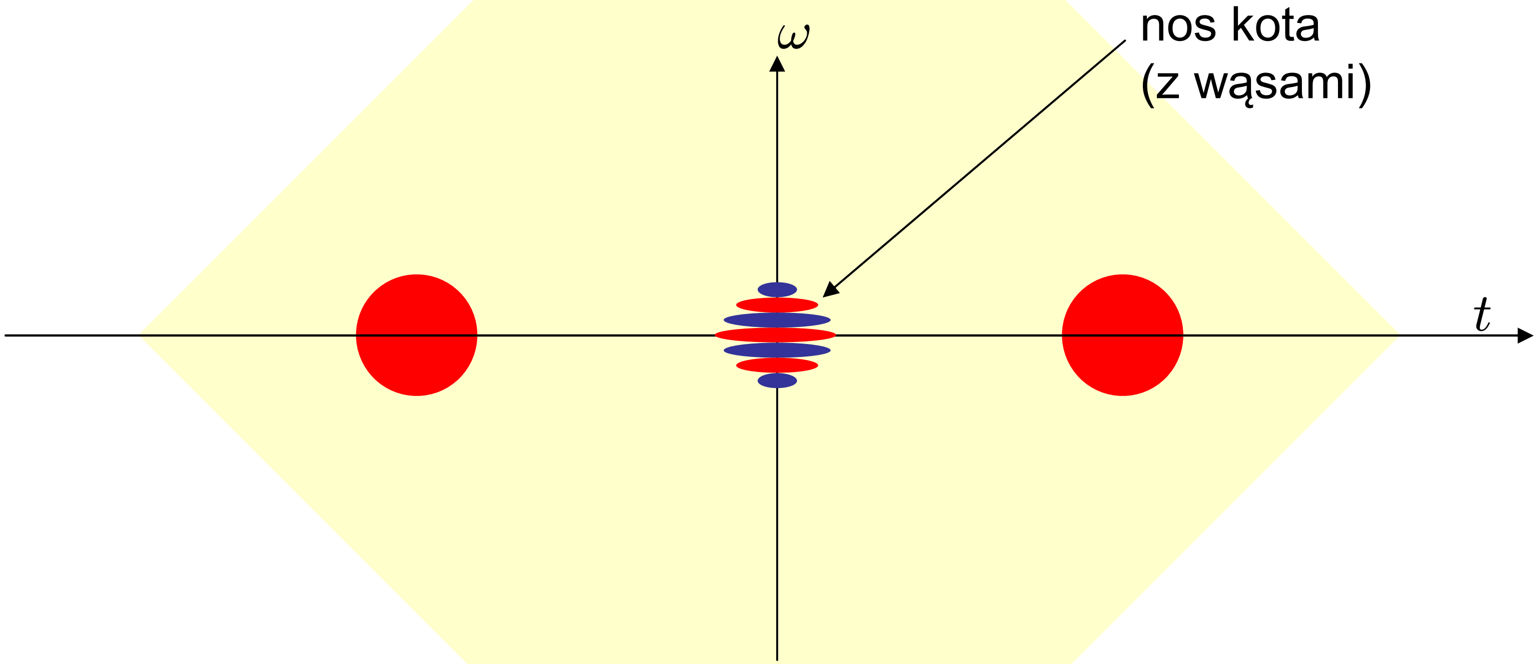


$$I(t) = \langle \alpha(t)^* \alpha(t) \rangle$$

$$\tilde{I}(\omega) = \int d\tau dt \langle \alpha(t_1)^* \alpha(t_2) \rangle e^{-i\omega(t_2 - t_1)}$$

$$W(\omega, t) = \frac{1}{\pi} \int d\tau \langle \alpha^*(t+\tau)\alpha(t-\tau) \rangle e^{-2i\omega\tau}$$

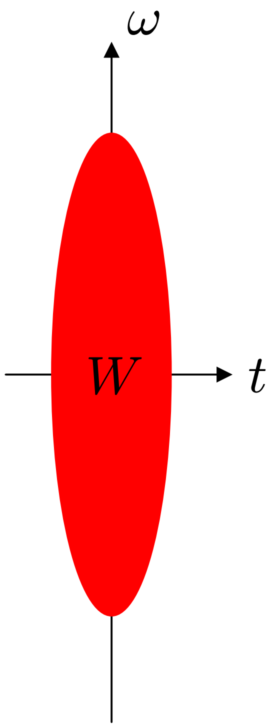
Wynik:
funkcja wignera dwóch impulsów



$$W(\omega, t) = \frac{1}{\pi} \int d\tau \langle \alpha^*(t+\tau)\alpha(t-\tau)\rangle e^{-2i\omega\tau}$$

Funkcja Wignera

$$W(\omega, t) = \frac{1}{\pi} \int d\tau \langle \alpha^*(t+\tau)\alpha(t-\tau)\rangle e^{-2i\omega\tau}$$

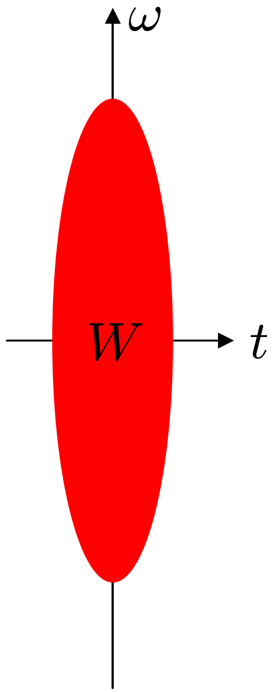


$$I(t) \propto |\alpha(t)|^2 = \int d\omega W(\omega, t)$$

$$\tilde{I}(\omega) \propto |\alpha(\omega)|^2 = \int dt W(\omega, t)$$

W przestrzeni częstości

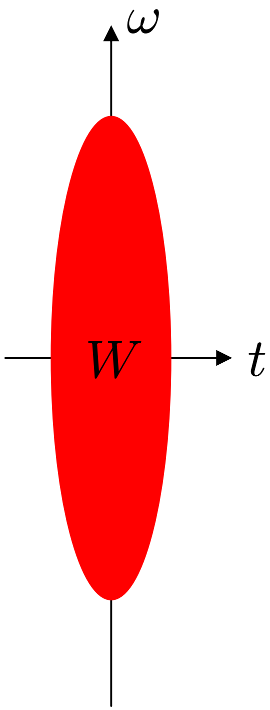
$$\alpha(t) = \int d\omega e^{-i\omega t}\tilde{\alpha}(\omega)$$



$$W(\omega,t) = \frac{1}{\pi}\int d\tau \langle \alpha^*(t+\tau)\alpha(t-\tau)\rangle e^{-2i\omega\tau}$$

$$W(\omega,t) = \frac{1}{\pi}\int ds \langle \tilde{\alpha}^*(\omega+s)\tilde{\alpha}(\omega-s)\rangle e^{\cdots ist}$$

Propagacja f. Wignera - dyspersja

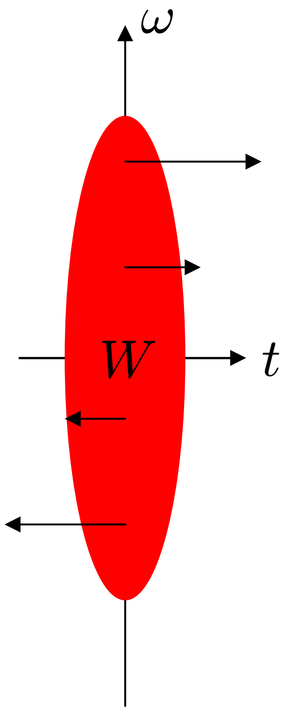


$$W(\omega,t)=\frac{1}{\pi}\int d\tau\langle\alpha^*(t+\tau)\alpha(t-\tau)\rangle e^{-2i\omega\tau}$$

$$\frac{\partial\alpha(t)}{\partial z}=i\beta_2\frac{\partial^2\alpha(t)}{\partial t^2}$$

$$\frac{\partial W(\omega,t)}{\partial z}=$$

Propagacja f. Wignera - dyspersja

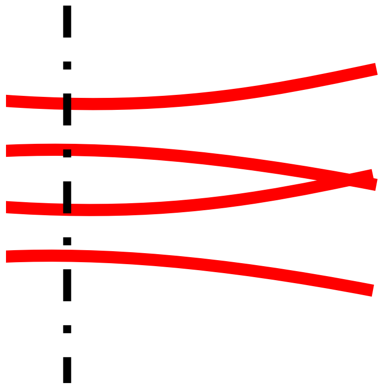


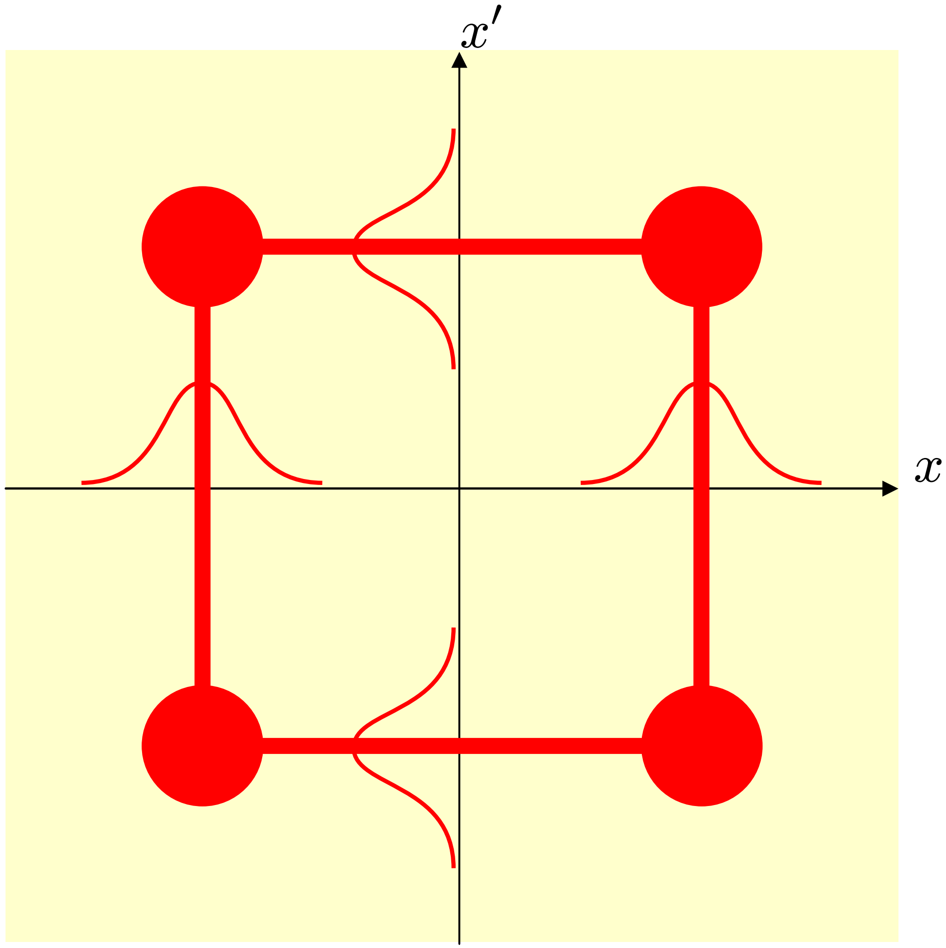
$$W(\omega,t)=\frac{1}{\pi}\int ds\langle\tilde{\alpha}^*(\omega+s)\tilde{\alpha}(\omega-s)\rangle e^{2ist}$$

$$\tilde{\alpha}(\omega)\to\exp\left(i\beta_2 z\omega^2\right)\tilde{\alpha}(\omega)$$

$$x\to x-z\frac{k}{k_0}$$

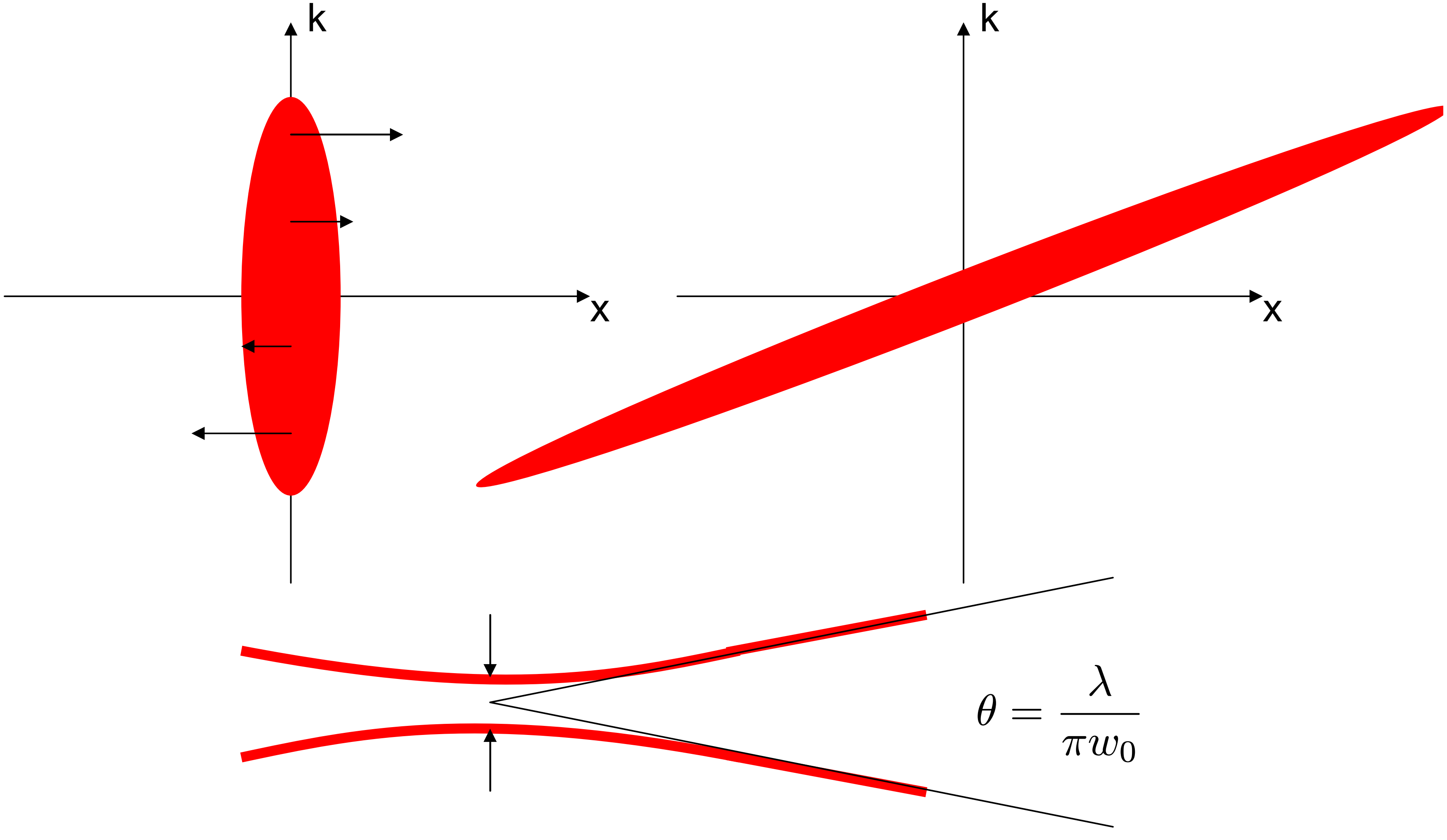
Przestrzenna funkcja wignera





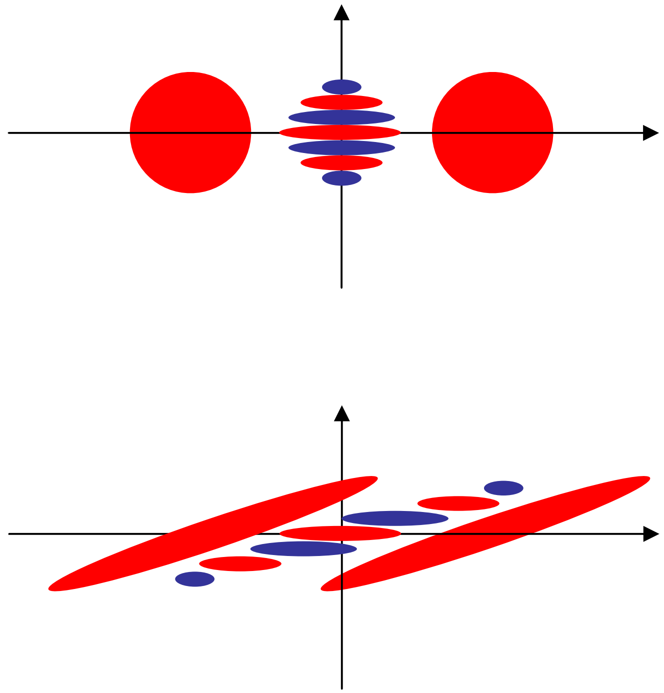
$$W(k,x) = \frac{1}{\pi}\int d\xi \langle \alpha^*(x+\xi)\alpha(x-\xi)\rangle e^{2ik\xi}$$

Funkcja Wignera



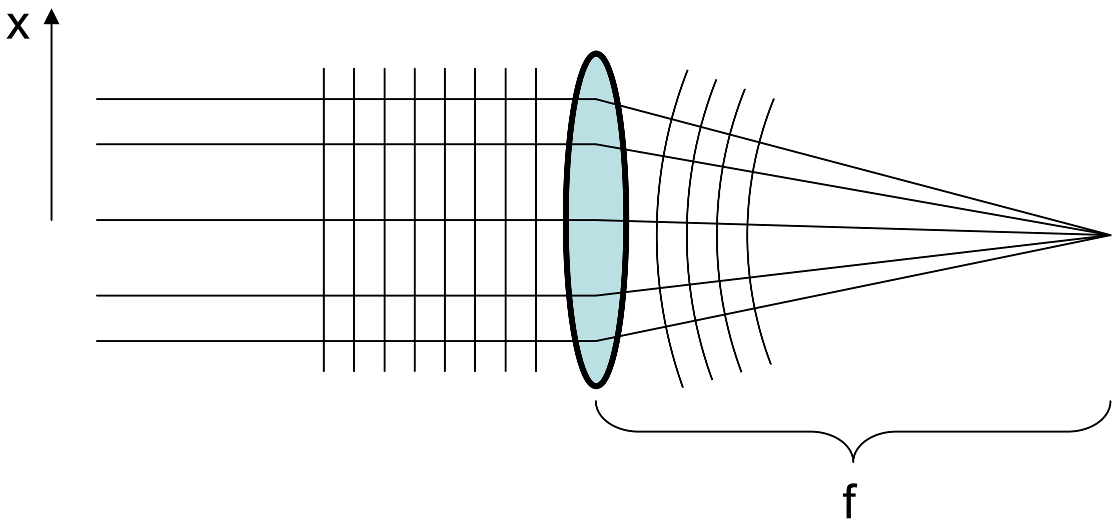
$$\theta = \frac{\lambda}{\pi w_0}$$

Znaczenie nosa



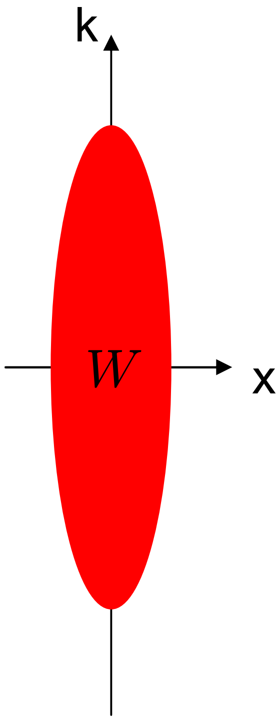
opóźnienie fazowe jednej wiazki?

Soczewka



$$\Delta\phi(x) = ik_0\left(f - \sqrt{f^2 - x^2}\right) \simeq ik_0\frac{x^2}{2f}$$

Propagacja f. Wignera - soczewka

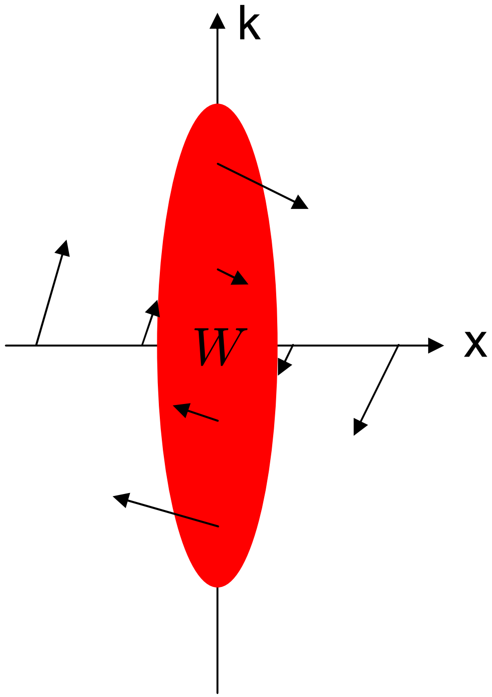


$$W(k,x) = \frac{1}{\pi}\int d\xi \langle \alpha^*(x+\xi)\alpha(x-\xi)\rangle e^{2ik\xi}$$

$$\alpha'(x) = \exp\left(\frac{ik_0}{2f}x^2\right)\alpha(x)$$

$$W'(k,x) =$$

Propagacja f. Wignera – gradient-index fiber



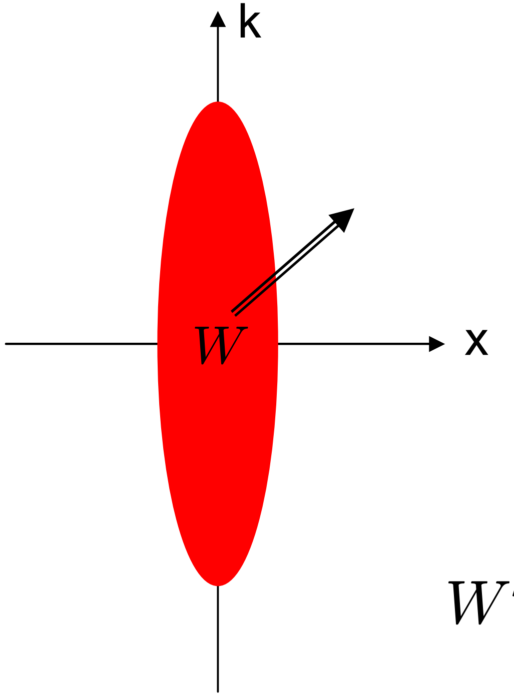
$$W(k,x) = \frac{1}{\pi}\int d\xi \langle \alpha^*(x+\xi)\alpha(x-\xi)\rangle e^{2ik\xi}$$

$$\frac{\partial \alpha(x)}{\partial z} = i\beta_2 \frac{\partial^2 \alpha(x)}{\partial x^2} + i\kappa x^2 \alpha(t)$$

$$\frac{\partial W(k,x)}{\partial z} =$$

jaki jest mod własny takiego światłowodu?

Przesunięcie i pchnięcie



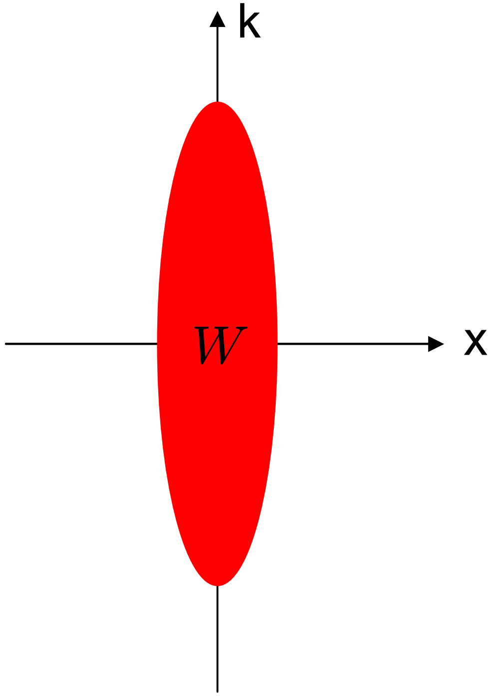
$$\alpha'(x) = e^{ik'x}\alpha(x+x')$$

$$= \underbrace{\exp\left(ik'x + ix'\frac{\partial}{\partial x}\right)}_{\hat{D}(k',x')}\alpha(x)$$

$$W'(k,x) = \frac{1}{\pi}\int d\xi \langle \alpha'^{*}(x+\xi)\alpha'(x-\xi)\rangle e^{2ik\xi}$$

jak się będzie propagować dowolna wiazka gaussowska
w światłowodzie gradient-index?

Symetria



$$W(0,0) = \frac{1}{\pi} \int d\xi \langle \alpha^*(\xi)\alpha(-\xi) \rangle$$

$$\hat{\Pi}\alpha(x) = \alpha(-x)$$

Cząstka kwantowa

$$W(q,p) = \frac{1}{\pi} \int dx \, \langle q - x | \hat{\rho} | q + x \rangle \, e^{2ipx}$$

$$p(q_0) = \langle \delta(\hat{q} - q_0) \rangle \qquad\qquad \delta(u) = \int \frac{d\xi}{2\pi} e^{i\xi u}$$

$$W(q,p) = \frac{1}{(2\pi)^2} \iint dk dx \, e^{-ikq - ixp} \langle e^{ik\hat{q} + ix\hat{p}} \rangle$$

Parzystość i operator przesunięcia

$$\hat{\Pi} = \int d\xi |-\xi\rangle\langle\xi|$$

$$\hat{D}(k,x) = e^{ik\hat{q}-ix\hat{p}}$$

$$\hat{D}(k,x)^{-1}\hat{q}\hat{D}(k,x) = \hat{q} + x$$

$$\hat{D}(k,x)^{-1}\hat{p}\hat{D}(k,x) = \hat{p} + k$$

$$\hat{\Pi}_{r,p} = \hat{D}(k,x)\hat{\Pi}\hat{D}(k,x)^{-1}$$

$$W(q,p) = \frac{1}{\pi}\int dx\, \langle q - x|\hat{\rho}|q + x\rangle\, e^{2ipx} \qquad \delta(u) = \int \frac{d\xi}{2\pi} e^{i\xi u}$$

$$p(q_0) = \langle \pm(\hat{q} - q_0)\rangle \qquad W(q,p) = \frac{1}{(2\pi)^2}\iint dk dx\, e^{-ikq-ixp}\langle e^{ik\hat{q}+ix\hat{p}}\rangle$$

Oscylator harmoniczny

$$W(q,p) = \frac{1}{\pi} \int dx \, \langle q - x | \hat{\rho} | q + x \rangle \, e^{2ipx}$$

$$\text{prob.}(q = q_0) = \langle \pm(\hat{q} - q_0) \rangle$$

$$W(q,p) = \frac{1}{(2\pi)^2} \iint dk dx \, e^{-ikq - ixp} \langle e^{ik\hat{q} + ix\hat{p}} \rangle$$