Zad. 1

$d = 1\,cm$

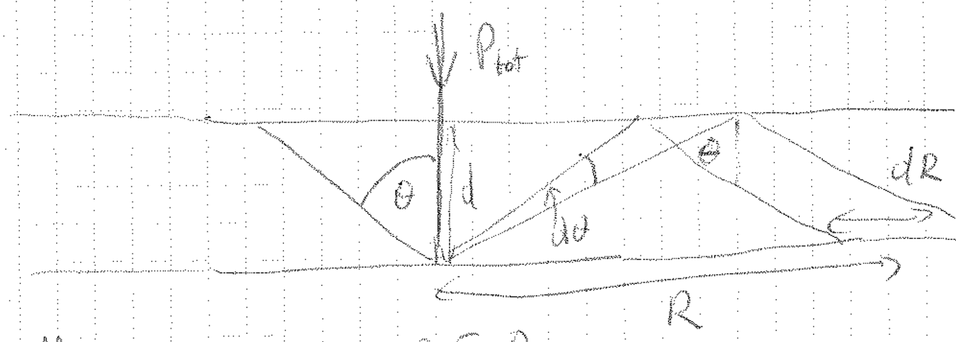
$P_{tot} = 1\,mW$

Kąt graniczny:

$n \sin\theta_g = 1$

$\theta_g = \arcsin 1/n$

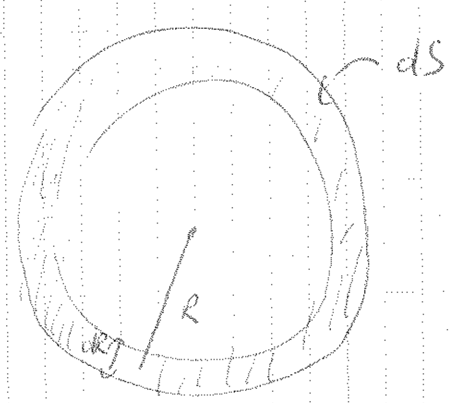
$R_g = 2d/\sqrt{n^2-1} = 2{,}28\,cm$



Moc rozproszona: $0.5\,P_{tot}$

Moc na jednostkę kąta bryłowego $\frac{dP}{d\Omega} = const = \frac{0.5\,P_{tot}}{2\pi}$

Natężenie: $I = \frac{dP}{dS}$



$$\frac{R}{2d} = \operatorname{tg}\theta \implies \cos\theta = \left(1 + \frac{R^2}{4d^2}\right)^{-1/2}$$

$$dS = 2\pi R\,dR = 8\pi d^2 \frac{\sin\theta}{\cos^3\theta} \quad \text{bo} \quad \frac{dR}{d\theta} = \frac{2d}{\cos^2\theta}$$

$$d\Omega = 2\pi \sin\theta\, d\theta$$

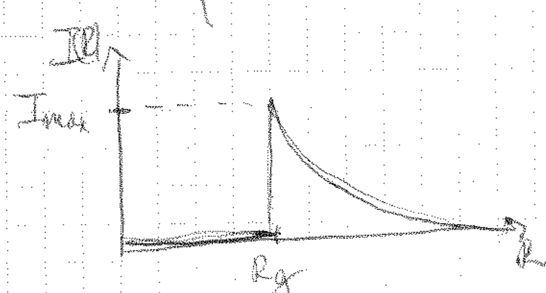
$I(R) = ?$

$$I(\theta) = \frac{dP}{dS} = \frac{dP}{d\Omega}\frac{d\Omega}{dS} = \frac{\frac{dP}{d\Omega}\frac{d\Omega}{d\theta}}{\frac{dS}{d\theta}} =$$

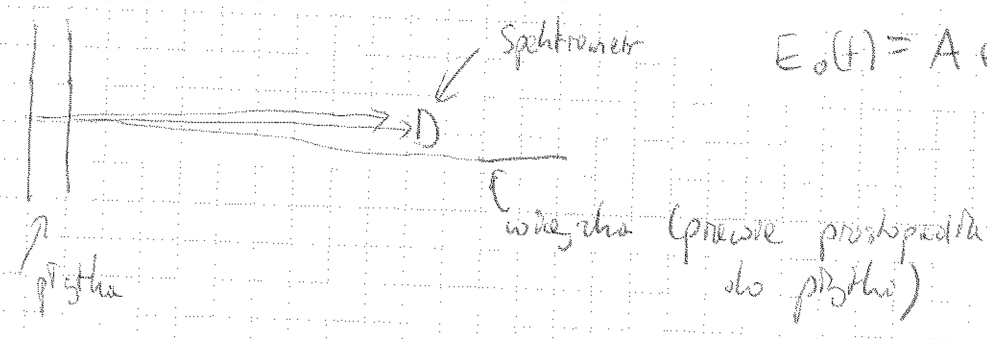
$$= \frac{\frac{0.5\,P_{tot}}{2\pi} \cdot 2\pi \sin\theta}{8\pi d^2 \frac{\sin\theta}{\cos^3\theta}} = \frac{1}{16\pi d^2} P_{tot} \cos^3\theta$$

$$I(R) = \begin{cases} \frac{P_{tot}}{16\pi d^2} \frac{1}{(1 + R^2/4d^2)^{3/2}} & \text{dla } R > R_g = 2d/\sqrt{n^2-1} \\ 0 & \text{dla } R \in (0, R_g) \end{cases}$$

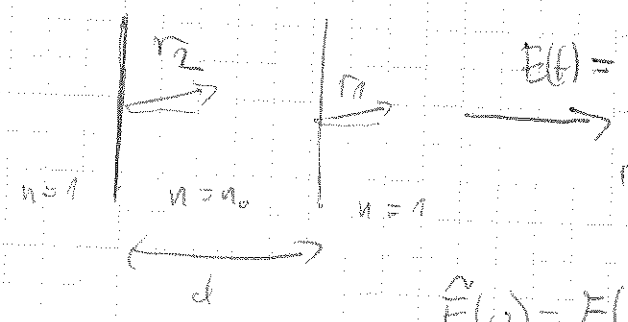
$$I_{max} = I(\theta_g) = \frac{P_{tot}}{16\pi d^2} \cos^3\theta_g = 5{,}8\,\frac{\mu W}{cm^2}$$



zad. 2



$$E_0(t) = A e^{-t^2/2\tau^2} \cos\omega_0 t$$

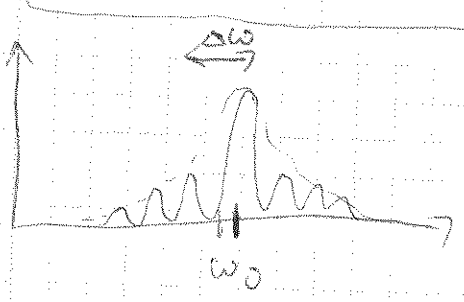


$$E(t) = r_1 E_0(t) + r_2 E_0\left(t + \frac{2dn_0}{c}\right)$$

$$r_1 = \frac{1-n_0}{1+n_0}, \quad r_2 = \frac{n_0-1}{n_0+1} \qquad r = r_2 = -r_1 = 0.2$$

$$\hat{E}(\omega) = \mathcal{F}(E(t)) = r\tilde{E}(\omega)\left(e^{-i\omega\frac{2dn_0}{c}} - 1\right)$$

$$\boxed{|\hat{E}(\omega)|^2 = r^2 |\tilde{E}(\omega)|^2 \sin^2\frac{dn_0}{c}\omega}$$



$$\Delta\omega = \frac{\pi c}{dn_0}$$

$$d = \frac{\pi c}{\Delta\omega\, n_0}$$

$\delta\lambda$ – rozdzielczość spektrometru

$$\omega = \frac{2\pi c}{\lambda} \qquad \delta\omega = -\frac{2\pi c}{\lambda^2}\delta\lambda$$

$$\Delta\omega > |\delta\omega| \Rightarrow \frac{\pi c}{dn_0} > \frac{2\pi c}{\lambda_0^2}\delta\lambda \Rightarrow d < \frac{\lambda_0^2}{2n_0\delta\lambda}$$

$$\Delta\omega \lesssim 1/\tau \qquad \frac{\pi c}{dn_0} \lesssim 1/\tau \qquad d \gtrsim \frac{\pi c \tau}{n_0}$$

Dla $n_0 = 1.5$, $\tau = 10\,fs$, $\lambda_0 = 800\,nm$, $\delta\lambda = 0.5\,nm$

$d \gtrsim 6\,\mu m$

$d < 430\,\mu m$

Zad. 3

$$\chi(\omega) = -\frac{\omega_p^2}{\omega^2}$$

$$n(\omega) = \sqrt{1-\frac{\omega_p^2}{\omega^2}}$$

$$k(\omega) = \frac{n(\omega)\,\omega}{c} = \frac{1}{c}\sqrt{1-\frac{\omega_p^2}{\omega^2}}\cdot\omega = \frac{1}{c}\sqrt{\omega^2-\omega_p^2}$$

$$V_f = \frac{\omega}{k(\omega)} = c\,\frac{\omega}{\sqrt{\omega^2-\omega_p^2}}$$

$$V_g = \frac{\partial\omega}{\partial k} = \left(\frac{\partial k}{\partial\omega}\right)^{-1} = c\left[\frac{2\omega}{2\sqrt{\omega^2-\omega_p^2}}\right]^{-1}$$

$$V_g = c\,\frac{\sqrt{\omega^2-\omega_p^2}}{\omega}$$

$$V_g\cdot V_f = c^2\,\frac{\omega}{\sqrt{\omega^2-\omega_p^2}}\cdot\frac{\sqrt{\omega^2-\omega_p^2}}{\omega} = c^2 = \text{const.}$$

$\blacksquare$